Review For Math 155 Exam 1

The directions for the exam are as follows:

"WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!"

1. In other words, the exam consists of 10 core problems and 2 extra-credit problems. If you wish, you can do all the 12 problems, but your score will only add up to 100 points. Partial credit will be given.

2. Also remember that you are allowed to use a scientific calculator.

3. When you are studying for this exam, be sure to work through sections that you know least of all first.

Warning! Be sure to work on ALL exercises listed below.

Section 5.1

- Given a function $f: A \to B$. Is f^{-1} always defined? Is it always a function? What is the domain of f^{-1}? What is the range?
- Explain what is a function and what is a multifunction.
- True or False? If $f: A \to B$ is injective (1-to-1), then $f^{-1}: B \to A$ is a well-defined function. If false, what must be modified?
- True or False? If $f: A \to B$ is a differentiable 1-to-1 function then $f^{-1}: f(A) \to A$ is also differentiable. Explain.
- What is the relationship between the graph of $y = f(x)$ and the graph $f^{-1}(y) = x$? How about $y = f(x)$ and $y = f^{-1}(x)$?
- Be able to determine whether a given function is injective. (**P. 259-260, Exercises 3-13 [odd]**).
- Play this simple game (**P. 260, Exercises 15-18**)
- Find a formula for the inverse function (**P. 260, Exercises 21-26**)
- Be able to compute the derivatives of inverse functions. (**P. 260-261, Exercises 37, 39, 41-44**).
- **Possible Extra-Credit:** If f is a 1-to-1, twice differentiable function with inverse function g. Find a formula for $g''(x)$.

Section 5.2

- Explain intuitively why we would expect the function $\ln(x)$ defined by $\ln(x) = \int_1^x \frac{1}{t} dt$ to be a logarithm. Specifically, differentiate $f(x) = a^x$ by definition and observe that $f'(x) = c(a)a^x$, where $c(a) = \lim_{h\to 0} \frac{a^h - 1}{h} = f'(0)$. Does f have an inverse function? Why? What is the derivative of f^{-1}? For what value of a is this derivative simplest?

- Use calculus to show that $\ln(x)$ does indeed have logarithmic properties such as $\ln(xy) = \ln(x) + \ln(y)$, etc.
- Use the laws of logarithms to expand the quantity/ express it as a single logarithm. (**P. 268, Exercises 1-7 [odd]**).
- Find the limit (**P. 268, Exercises 13, 14**).
- Differentiate. (**P. 268, Exercises 15-31 [odd]**).
- Find y''. (**P. 268, Exercises 33, 34**).
- Be able to recognize when logarithmic differentiation is handy (**P. 269, Exercises 51-54**).
- Evaluate integrals (**P. 269, Exercises 55-61 [odd]**). Note that one should write $\int \frac{1}{x} dx = \ln(|x|) + C$ rather than $\ln(x) + C$. Why?

Section 5.3

- How can we define the number e using the properties of $\ln(x)$? Explain why $\ln(e^x) = x$.
- Explain in as many ways as you can why $\frac{d}{dx} e^x = e^x$.
- Simplify each expression (**P. 274, Exercises 2-4**).
- Solve for x. (**P. 274, Exercises 5-8**).
- Find the limit. (**P. 274, Exercises 17-22**).
- Differentiate (**P. 274, Exercises 23-37 [odd]**).
- Evaluate the integral (**P. 275, Exercises 61-67 [odd]**).
- **Possible Extra-Credit:** If $f'(x) = f(x)$, what is the relationship between $f(x)$ and e^x? Repeat under the assumption $f'(x) = kf(x)$.
- **Possible Extra-Credit:** Show that $e = \lim_{h\to 0}(1 + h)^{1/h}$.
- **Possible Extra-Credit:** Argue that the conditions $f'(x) = f(x)$ and $f(0) = 1$ imply that $f(x) = \lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n$.
- **Possible Extra-Credit:** Is $f(x) = e^x$ a finite polynomial? Why or why not? Express $f(x)$ as an infinite polynomial $\sum_{n=0}^{\infty} a_n x^n$. Find the coefficients.
- **Possible Extra-Credit:** Express $\cos x$ and $\sin x$ as infinite polynomials. Do you see any interesting relationships between these trigonometric functions and e^x?

Section 5.4

- Write the expression as a power of e. (**P. 282, Exercises 3-6**).
- Evaluate expression (**P. 282, Exercises 7-10**).
- Find the limit. (**P. 282, Exercises 21-22**).
- Differentiate. (**P. 282, Exercises 23-37 [odd]**).
- Evaluate the integral. (**P. 283, Exercises 41-46**).

Section 5.5

- Be able to solve exponential growth\ decay problems. (**P. 289-290, Exercises 3, 7, 9, 13, 15, 19**).

Section 5.6

- Observe that since $\cos\quad, \sin\quad : \mathbb{R} \to [-1,1]$ are periodic, $\cos^{-1}\quad, \sin^{-1}\quad : [-1,1] \to \mathbb{R}$ are multifunctions of the form 1-to-∞. How do we define the principal branches of $\cos^{-1}$ and $\sin^{-1}$? In other words, how do we restrict $\sin$ and $\cos$ to make their inverses proper functions?
- What is the domain and range of the six inverse functions? Compute their derivatives.
- Find the exact value of each expression. (**P. 297, Exercises 1-10**).
- Verify the given identities. (**P. 297, Exercises 11-15**).
- Differentiate. (**P. 297, Exercises 17-33 [odd]**).
- **Possible Extra-Credit:** (**P. 297, Exercises 37, 38**).
- Integrate (**P. 297-298, Exercises 39-47 [odd]**).

Section 5.8

- l'Hospital's rule is nothing more than recognizing the definition of derivative scrambled within a limit. (**P. 311, Exercises 1-39 [odd]**).
- Be sure to look through the limit problems on the Review Of Math 150 worksheet.
- **Possible Extra-Credit:** Suppose that some function f has the following properties. $f(x+y) = f(x) + f(y) + x^2y + xy^2$ and $\lim_{x\to 0} \frac{f(x)}{x} = 5$. Find $f'(x)$ for any value x.
- **Possible Extra-Credit:** Evaluate $\lim_{x\to\infty}\left(x - x^2 \ln\left(\frac{1+x}{x}\right)\right)$.

Section 6.1

- Be able to apply Integration By Parts (i.e. Reverse Product Rule) to solve various integral problems. (**P. 322, Exercises 3-29 [odd]**).
- Use integration by parts to prove the reduction formulas. (**P. 323, Exercises 35-38**).

Section 6.2

- Be able to evaluate integrals involving trigonometric functions or requiring trigonometric substitution. (**P. 332, Exercises 1-59 [odd]**).